Serviceability limit states of wooden footbridges
Vibrations caused by pedestrians

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1 General

This paper deals with EN 1995-2:2004 (E) – Eurocode 5: Design of timber structures, Part 2: Bridges, from 2004 [1]. In combination with the corresponding German National Annex [2] it became a German standard in 2010.

The comfort criteria should be taken from EN 1990:2002/A1 [3].

The verifications of vibrations of wooden footbridges caused by pedestrians in the serviceability limit state are explained and discussed.

Depending on the natural frequencies of the bridge the following verifications of the bridge acceleration are recommended in the informative Annex B "Vibrations caused by pedestrians" of [1] and in the German National Annex [2], see figure 1. a_{vert} and a_{hor} are the resulting accelerations of the bridge in the vertical and horizontal direction. If all vertical natural frequencies of the bridge are greater than 5 Hz and all horizontal natural frequencies are greater than 2,5 Hz no further verifications of the vibrations are needed.

Recognize that all these verifications are in the serviceability limit state. Although these criteria and equation are presented in a norm for timber structures EC5 [1], they are valid for all materials, with exception of the damping factor.

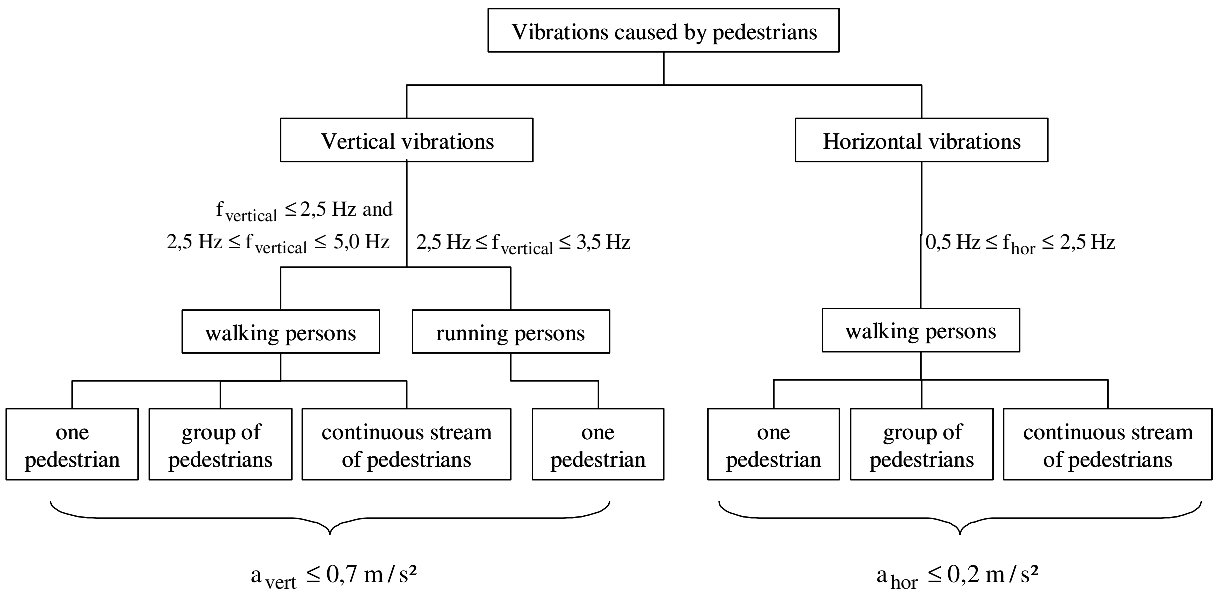


Figure 1: Verifications of bridge vibrations caused by pedestrians recommended in [1], [2], [3]

2 Bridge acceleration due to passing pedestrians

2.1 Natural frequency

To get the natural frequencies of the bridge, one can use a finite elemente programme or transfer the bridge into a dynamic modell with mass M* and stiffness K*, see figure 2.

$$f_{vert} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \cdot \sqrt{\frac{K^*}{M^*}} \text{, neglecting the damping of the bridge R} \quad (1)$$

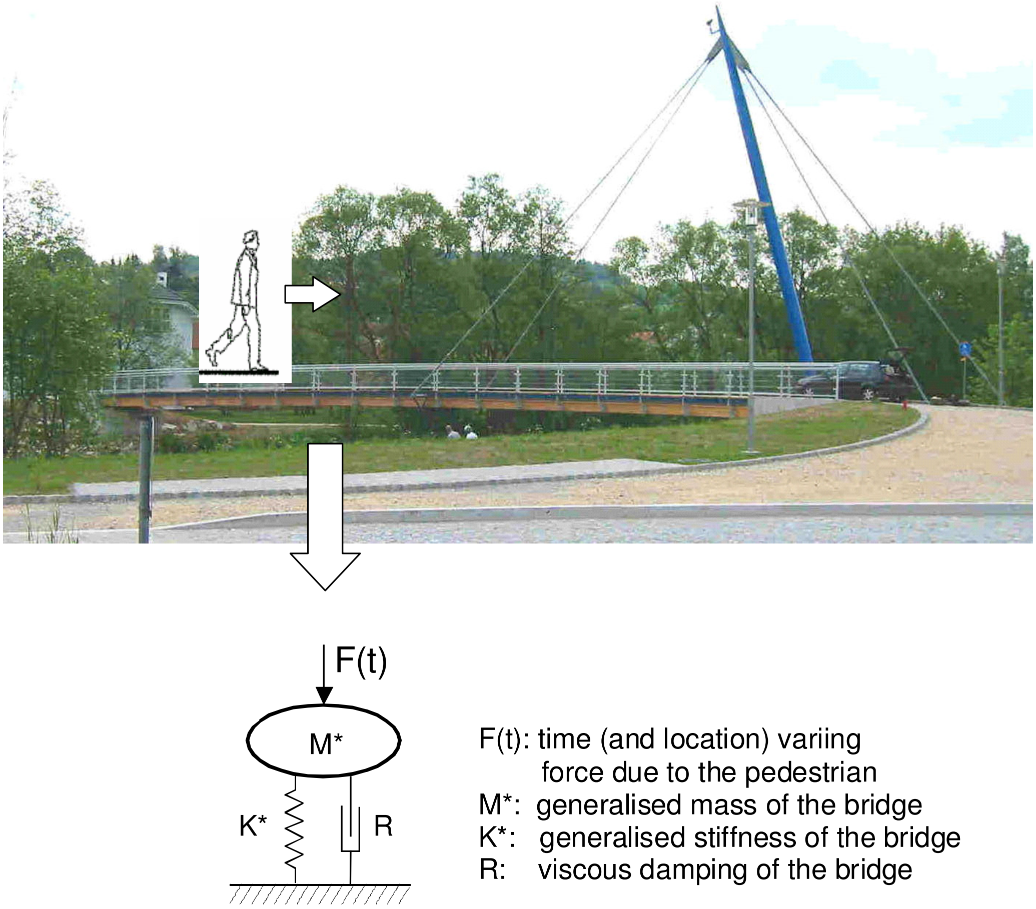


Figure 2: Footbridge in Zwiesel: Picture and dynamic modell of the bridge incl. pedestrian

2.2 Dynamic load caused by pedestrians

The pedestrian crossing the bridge is transferred into a dynamic load F(t). The graph of the function of F(t) for walking and running is shown in figure 3 (thick line) as the sum of the first three Fourier coefficients (thin lines). In practice only one of the Fourier coefficients is relevant – the one closest to the natural frequency. For the Fourier decomposition see [4].

Table 1 shows the step frequencies and the first harmonic parts of the load depending on the kind of movement, with $F_0 \approx 700N$ = weight of the pedestrian.

Factor k in table 1 considers the fact, that the pedestrian is crossing the bridges and not acting on the same place all the time. It ranges between (0,4 …) 0,6 … 0,75 (… 0,8), depending on the static system and the damping of the bridge [5].

	step frequency f_s	**vertical load on bridge**	**horizontal load on bridge**
walking	1,5 … 2,5 Hz	$k \cdot 0{,}4 \cdot F_0$	$k \cdot 0{,}1 \cdot F_0$
running	2,5 … 3,5 Hz	$k \cdot 1{,}3 \cdot F_0$	$k \cdot 0{,}1 \cdot F_0$

Table 1: Step frequencies and 1st harmonic parts of the load for walking and running

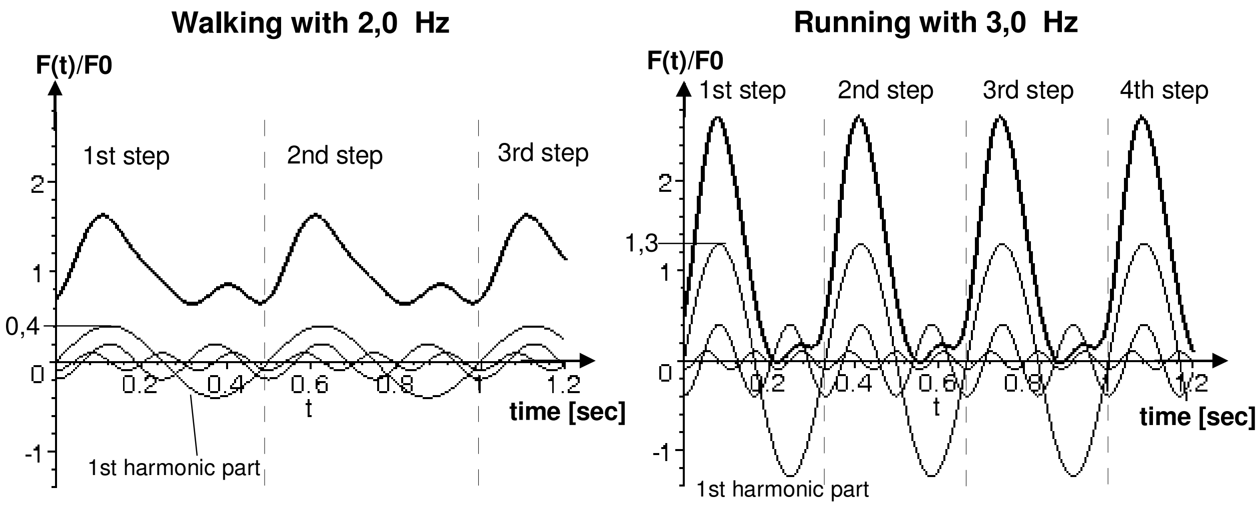


Figure 3: Graphs of the loads due to a walking and running pedestrian (total (thick line) and the first 3 harmonic parts (thin lines))

2.3 Acceleration caused by pedestrians

When the load on the bridge is known, the answer of the bridge can be calculated. Equation 3 gives the maximum value of the acceleration of the bridge, assuming resonance between bridge- and step frequency (because it is the worst case).

$$F_{vert,walking}(t) = k \cdot 0{,}4 \cdot F_0 \cdot \sin(2\pi \cdot f_s) \approx 200 \cdot \sin(2\pi \cdot f_s) \qquad (2a)$$

$$F_{vert,running}(t) = k \cdot 1{,}3 \cdot F_0 \cdot \sin(2\pi \cdot f_s) \approx 600 \cdot \sin(2\pi \cdot f_s) \qquad (2b)$$

$$a_{vert,1,walking} = \frac{200}{M^* \cdot 2\zeta} \qquad (3a)$$

$$a_{vert,1,running} = \frac{600}{M^* \cdot 2\zeta} \qquad (3b)$$

ζ is the damping factor (see chapter 3). The relation between the damping factor ζ and the viscous damping R is shown in equation 4 (for ω see equation 1):

$$R = 2 \cdot \zeta \cdot M^* \cdot \omega \qquad (4)$$

The equation 3a, b (taken from [1] and [2]) are a good approximation to calculate the bridge acceleration due to one pedestrian, see figure 6. The user of the equation needs some experience in calculating the generalized mass of the bridge, or one can use a finite element programme therefore.

When using such a programme, one can perform a more detailed calculation with a time and location varying load. Depending on the natural frequencies of the bridge walking and/or running pedestrians should be considered (see table 1 and figure 3). The load velocity along the bridge is $0{,}9\mathrm{m} \cdot f_s$, factor k is 1.

The load- acceleration- relation in the horizontal direction is a little bit different. This is because of the fact, that only each second step gives a load to the right, and the other ones to the left. The load frequency is half of the step frequency, see equation 5. For the maximum value of the horizontal acceleration see equation 6.

$$F_{hor,walking}(t) = k \cdot 0{,}1 \cdot F_0 \cdot \sin\left(2\pi \cdot \frac{1}{2} \cdot f_s\right) = 50 \cdot \sin\left(2\pi \cdot \frac{1}{2} \cdot f_s\right) \tag{5}$$

$$a_{hor,1,walking} = \frac{50}{M^* \cdot 2\zeta} \tag{6}$$

Equations 3a, b and 6 are given in Annex B (informative) in [1]. They give the acceleration due to one pedestrian in resonance.

What should one do to consider more than one pedestrian? Are they all acting in resonance? In [8] a study is described to get a synchronization factor depending on the natural frequency of the bridge. If the natural frequency is in the range of the usual step frequency for walking (see table 1), the synchronization factor is 0,23 for vertical vibrations and 0,18 for horizontal vibrations. This factor takes care of the fact that pedestrians are likely to fall into the frequency of the bridge vibration.

With help of this factor the acceleration due to a group or a continuous stream of pedestrians can be calculated.

$$a_{vert,group,walking} = 0{,}23 \cdot 13 \cdot k_{vert} \cdot a_{vert,1,walking} \tag{7a}$$

$$a_{hor,group,walking} = 0{,}18 \cdot 13 \cdot k_{hor} \cdot a_{hor,1,walking} \tag{7b}$$

13 is the number of pedestrians in the group.

$$a_{vert,stream,walking} = 0{,}23 \cdot (0{,}6 \cdot b \cdot \ell) \cdot k_{vert} \cdot a_{vert,1,walking} \tag{8a}$$

$$a_{hor,stream,walking} = 0{,}18 \cdot (0{,}6 \cdot b \cdot \ell) \cdot k_{hor} \cdot a_{hor,1,walking} \tag{8b}$$

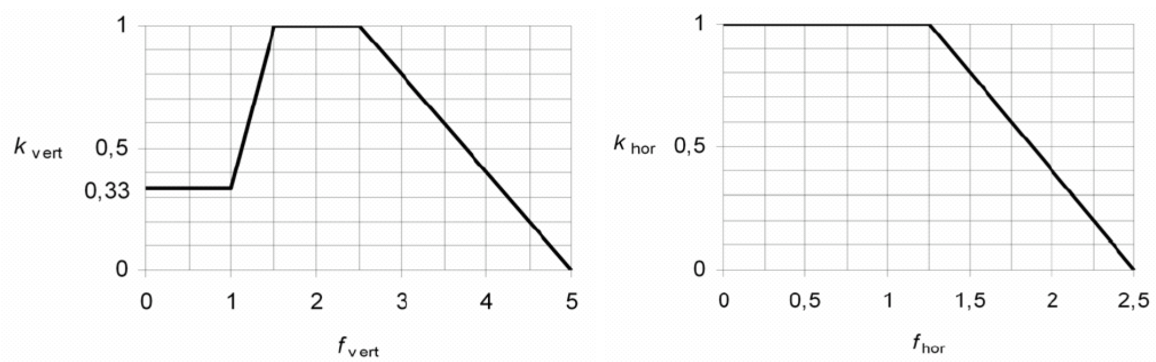
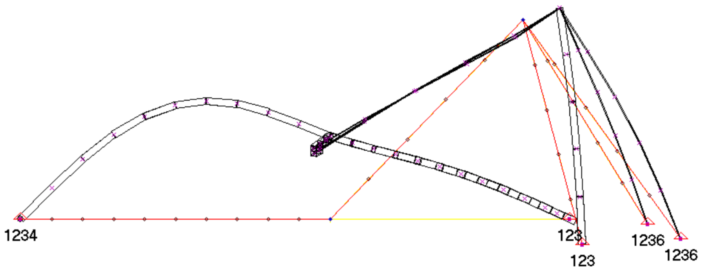


Figure 4a, b: Relationship between the vertical /horizontal natural frequency and the coefficient k_{vert} and k_{hor}, taken form [1]

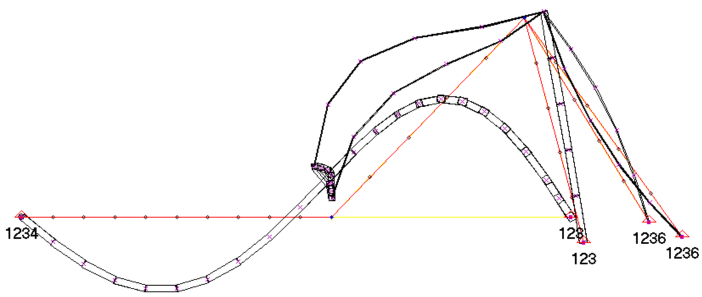
In equations 8a and 8b 0,6 pedestrians each square meter are assumed. b is the width of the bridge, ℓ the total length. The factors k_{vert} and k_{hor} (shown in figures 4a and 4b) consider, that the synchronization is possible only in the range of usual step frequencies.

2.4 Example: Bridge in Zwiesel across the creek Schwarzer Regen

This example shows the results of vibration measurements on a Bridge in Zwiesel, see figure 2. Before the bridge was built, the natural frequencies and the expected accelerations had been calculated, see figure 5a, b. In table 2 the results are shown. The calculated accelerations had been such high, so that the installation of a vibration damper was planned. The measurement confirmed the calculation and the necessity of the damper. A running group of 7 pedestrians reached an acceleration of 5,4 m/s²!



1st mode: f_e = 1,81 Hz



2nd mode: f_e = 2,58 Hz

Figure 5a, b: 1st and 2nd vertical bending modes and frequencies of the bridge in Zwiesel

	calculated	**measured**
first vertical natural frequency	1,78 Hz	1,81 Hz
second vertical natural frequency	2,41 Hz	2,58 Hz
acceleration due to a walking pedestrian	$\frac{200}{19750 \cdot 2 \cdot 0{,}01} = 0{,}50 \frac{m}{s^2}$	first f_e: 0,60 m/s² second f_e: 0,40 m/s²
acceleration due to a running pedestrian	$\frac{600}{19750 \cdot 2 \cdot 0{,}01} = 1{,}5 \frac{m}{s^2}$	second f_e: 1,70 m/s²

Table 2: Comparison of calculated and measured parameters of the bridge in Zwiesel

2.5 Further comparisons

Altogether more than 20 wooden footbridges have been examined. They have been excited by pedestrians walking and running in resonance. The measured accelerations have been compared with the calculated accelerations (see equations 3a and 3b, for the damping factor see table 3). The results are shown in figure 6.

The calculated and measured accelerations fit together in most of the cases. If the values do not fit, the calculation is on the "save side", this means, the calculated value is higher than the measured one.

There are some differences between the values, especially for the bridges with high natural frequencies. This is because of the fact, that running with step frequencies of 3,9 Hz and more is very difficult. In the other cases with "calculated accelerations are greater than measured accelerations", the runner probably was not exactly in resonance with the bridge.

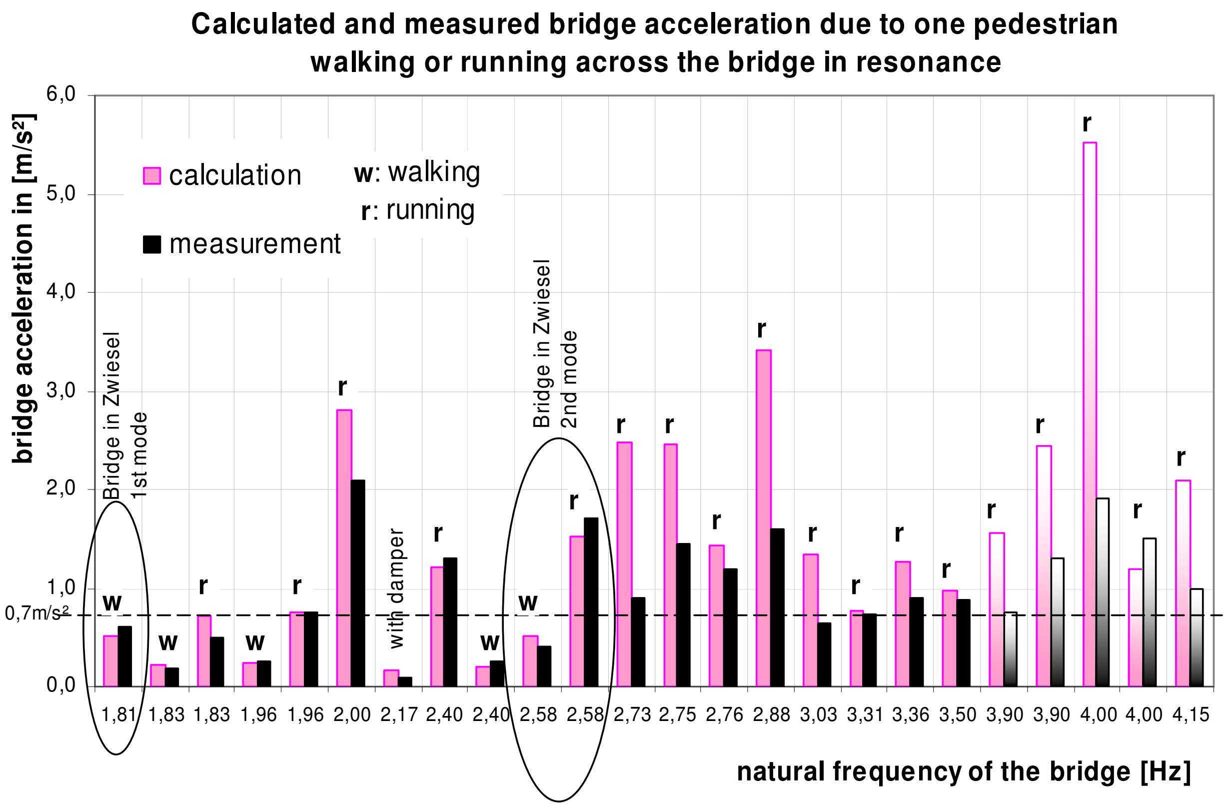


Figure 6: Comparison of calculated and measured bridge accelerations due to one pedestrian walking or running across the bridge in resonance

2.6 Discussion

As one can see in figure 6, most of the measured bridge accelerations due to a running pedestrian are greater than 0,7 m/s². With only few exceptions no bridge has a damper installed. So one can ask:

Is the requirement in Eurocode [3] and Eurocode 5 [1] to strict?

What should one do with the running group or stream?

In the authors opinion the limit of the acceleration caused by running people can be greater than for walking groups. Vibrations caused by running people are more likely accepted than vibrations caused by walking people. The (new) limit of the bridge acceleration caused by running people should depend on the situation of the bridge, for example: the using frequency, the location of the bridge, the static system (see table 3).

Criteria:	The bridge is …	
using frequency How often is the bridges used?	used rather often.	used rather seldom.
number of pedestrians	often used by crowds of pedestrians.	used by single persons only.
location of the bridge	located centrally in the city.	located away in the country.
	located as a viewpoint (a place to stay).	
static system	covered with a roof (a place to stay).	a simple static system (e.g. simple supported beam).
	suspended by cables (fatigue and optic problem).	
Consequence:	The limit of the bridge acceleration due to running pedestrians should …	
	be near the values given in figure 1.	be increased.

Table 3: Criteria to increase the acceleration limit for footbridges … or not

3 Damping factor

In EC5 [1], chapter 6.5.1 (2) the following information is given:

"(2) Where no other values have been verified, the damping ratio should be taken as:
- ζ=0,010 (=1,0%) for structures without mechanical joints,
- ζ=0,015 (=1,5%) for structures with mechanical joints.
NOTE 1: For specific structures, alternative damping ratios may be given in the National annex."

To specify the damping factor of wooden footbridges the following two methods were used.

3.1 Decreasing vibration

With a mechanical exciter a sinusoidal load is put on the bridge. Then the steady state vibration is reached, the excitation is stopped and the decreasing vibration can be measured (see figure 8b, c). The mechanical exciter in figure 7 was designed and constructed at the Technische Universität München, Fachgebiet Holzbau [5].

The damping factor can be calculated with equations 9 and 10.

$$\Lambda = \frac{1}{n} \cdot \ln \frac{a_0}{a_n}; \quad \zeta = \frac{\Lambda}{2\pi} \qquad (9); (10)$$

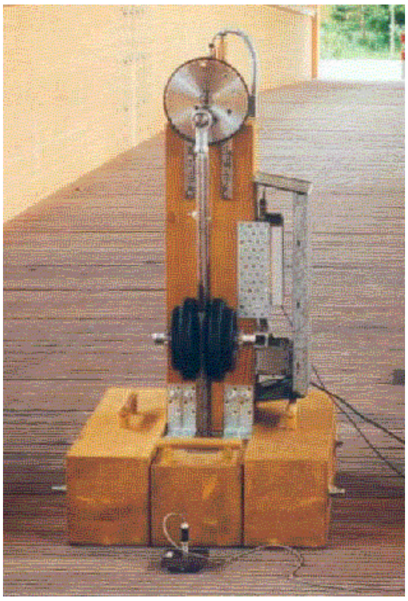


Figure 7: Mechanical exciter

Figure 8 shows two “extreme” damping behaviours of wooden footbridges, a bending beam bridge made of glued laminated timber and bituminous mastic concrete in Uttenreuth and a cable stayed bridge with long cables in Oberesslingen.

Figure 8a: Pictures of the footbridges in Uttenreuth and Oberesslingen

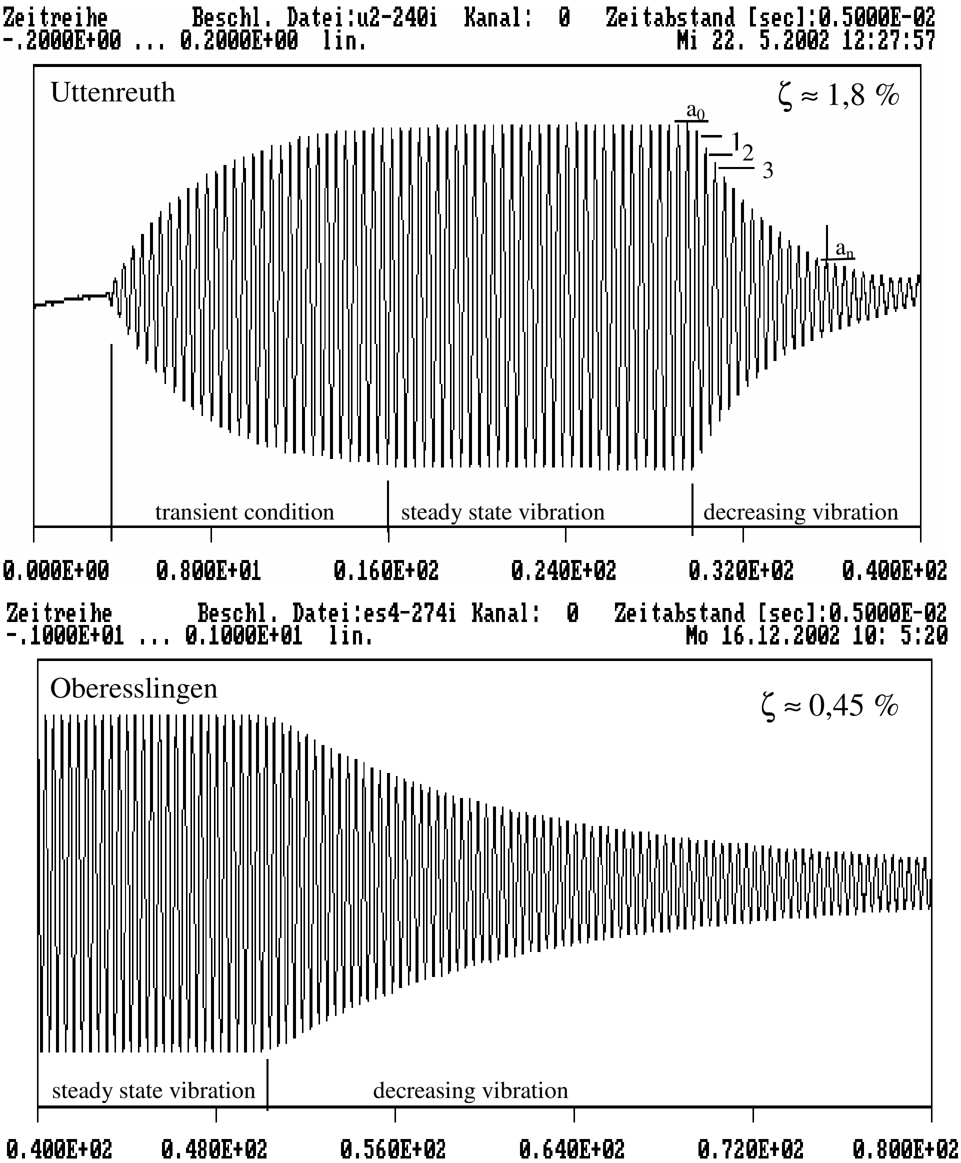


Figure 8b, c: Recordings of the measurements for Uttenreuth and Oberesslingen.

3.2 Resonance curve

The second method to measure and calculate the damping factor of a bridge deals with the resonance curve. Hereby the bridge is excited by the mechanical exciter with sinusoidal loads in different exciting frequencies. The amplitudes of the bridges in the steady state vibration due to the different loads are measured and the resonance curve can be drawn. The maximum amplitude a_{max} is at resonance between the natural bridge and the exciting frequency. Take $a_{max} / \sqrt{2}$ and calculate the corresponding exciting frequencies f_1 and f_2. The difference between these frequencies divided through 2 times the natural frequency is the damping factor (see equation 11 and figures 9 a, b).

$$\zeta = \frac{f_2 - f_1}{2 \cdot f_e} \qquad (11)$$

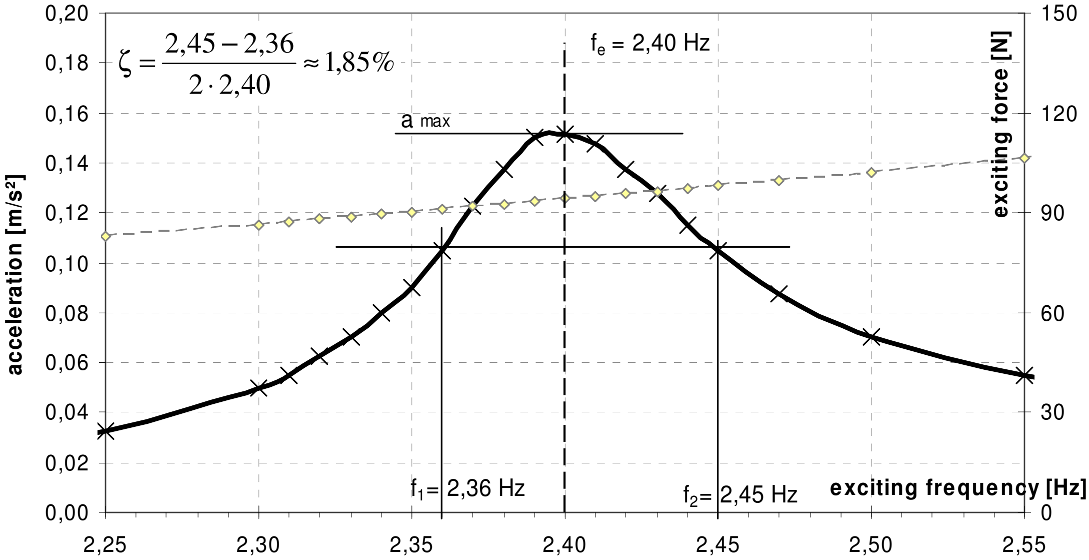


Figure 9a: Resonance curve (measured acceleration and exciting force) for a wooden footbridge in Uttenreuth.

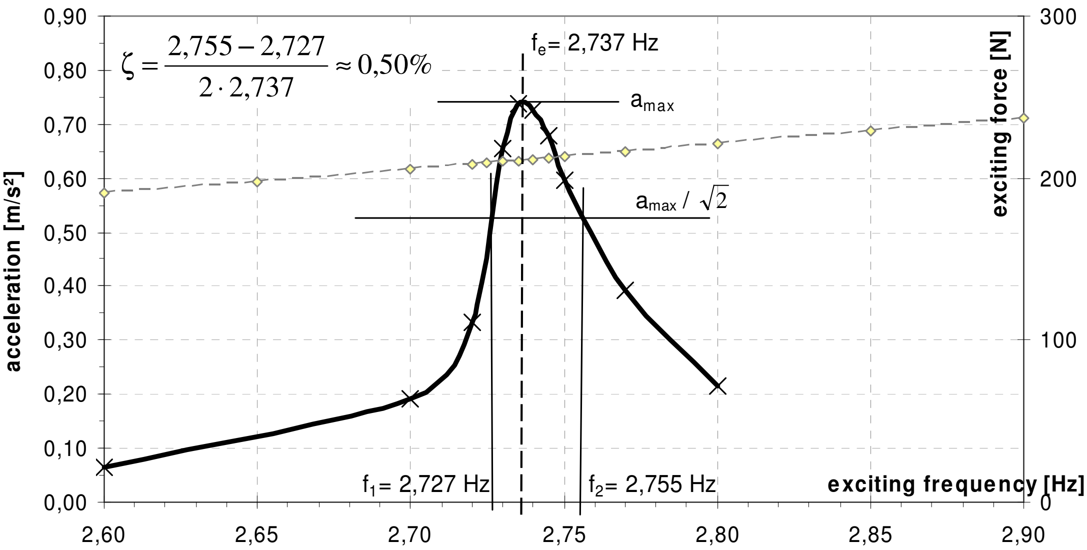


Figure 9b: Resonance curve (measured acceleration and exciting force) for a wooden footbridge in Oberesslingen.

3.3 Results for the damping factor

The damping factors of about 20 bridges have been measured by these two methods and sorted by their static systems. In table 4 suggestions for the damping factors are given.

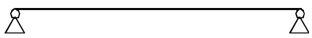
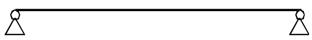
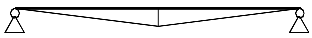
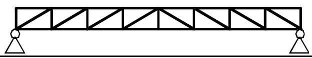
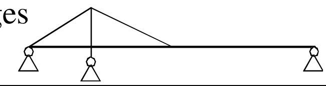
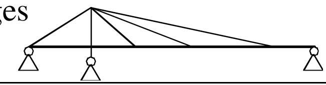
Static system of beam / bridge	**Damping factor**
Glued laminated beam 	0,50 %
Bending beam bridges made of glued laminated timber 	1,20 %
Suspended beam bridges 	0,90 %
Frame work bridges 	0,80 %
Cable stayed bridges with short cables 	1,00 %
Cable stayed bridges with long cables 	0,30 %
Bituminous mastic concrete	additional 0,30 %

Table 4: Suggestion for the damping factor of wooden beams and footbridges depending on their static system, which is exemplarily shown in the drawings

4. References

[1] Eurocode 5 - Design of timber structures, Part 2: Bridges. EN 1995-2: 2004 (E) and DIN EN 1995-2:2010-12 (EN 1995-2:2004 (D)).

[2] Eurocode 5: Bemessung und Konstruktion von Holzbauten, Teil 2: Brücken. DIN EN 1995-2 /NA: 2011-08. National Annex – Nationally determined parameters – Eurocode 5: Design of timber structures – Part 2: Bridges

[3] Eurocode – Basis of structural design/Amendment A1 – Annex A2: Application to Bridges. EN1990:2002/A1

[4] Bachmann, Hugo et al.: "Vibration Problems in Structures - Practical Guidelines". 2nd Edition, Birkhäuser Verlag Basel, Berlin, Boston, 1997.

[5] Hamm, Patricia: "Ein Beitrag zum Schwingungs- und Dämpfungsverhalten von Fußgängerbrücken aus Holz". Dissertation. TU München, November 2003.

[6] Hamm, Patricia: "Vibrations of wooden footbridges induced by pedestrians and a mechanical exciter". In: "footbridge 2002, Proceedings of the International Conference on the Design and dynamic behaviour of footbridges" Paris, 20. - 22. November 2002. Editor: AFGC and OTUA. pp 144-145.

[7] Kreuzinger, Heinrich: "Dynamic design strategies for pedestrian and wind actions". In: "footbridge 2002, Proceedings of the International Conference on the Design and dynamic behaviour of footbridges" Paris, 20. - 22. Nov. 2002. Editor: AFGC and OTUA. pp 129-141.

[8] Grundmann, Harry; Kreuzinger, H.; Schneider, M. "Schwingungsuntersuchungen für Fußgängerbrücken". In: Bauingenieur 68 /1993. pp 215-225.