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# MONITORING OF TIMBER BRIDGES BY REPEATEDLY MEASURING THE NATURAL FREQUENCY

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**ABSTRACT:** Measuring the natural frequency of buildings and bridges is a possibility to get information about the stiffness of the construction. Decreasing stiffness can be detected be repeatedly measurements. Damaged parts of the construction or too high wood moisture can be reasons for decreasing stiffness. The earlier the failure is detected the better is the chance to repair it with low costs. The method of monitoring by repeatedly measuring the natural frequency is applied at timber bridges, especially on footbridges. As damages due to high wood moisture cannot be seen easily, measuring the natural frequency is a good possibility to detect them and then to repair them. Equations to calculate the natural frequencies regarding the damaged parts are shown and applied to a simple supported beam.

**KEYWORDS:** measurements, natural frequency, timber bridges, footbridges, damages,

## 1 Introduction

Bridges are subjected to natural environment. Therefore the moisture content can increase locally, providing the necessary environment conditions for the growing of timber deterioration fungi. These critical points are often in the range of joints, which can hardly be checked visually (see Fig. 1).

Since the deterioration of timber leads to a reduction of the load capacity, the critical points in the structure have to be identified.
One possible method to identify these critical points is a proof loading of the system. Within this method the structure is loaded statically and the reaction is monitored e.g. via photogrammetry. Comparing the load with the reaction of the system, the deterioration of the system can be checked in principle.

However proof loading is quite expensive and time consuming, since the load has to be applied and the measurement equipment has to be installed.

a.) Bridge

b.) Damage in the range of the joint

***Figure 1:*** *Example of a bridge with a damage in the joint between planks and beam*

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The applied loads are often dead loads, which can be determined by

$$F = m \cdot a \qquad (1)$$

where $F$ force
$m$ mass
$a$ acceleration

In the static case, the acceleration is the gravity. In the dynamic case the acceleration covers the reaction of the system as well as the gravity. So additional forces arise if the system is loaded dynamically. One case of dynamic loading is, when vibrations in the system occur. These vibrations can be caused by temporally acting forces as wind or persons crossing the bridge. The system starts vibrating and therefore the masses of the structure are accelerated, leading to a force in the system.
Therefore a vibration can be interpreted as a temporarily acting proof load. The values of the eigenfrequencies depend on the mechanical properties of the structure. Changing mechanical properties as the deterioration of the cross section should be identified by the measurements of the eigenfrequency. In the following a concept will be proposed, with which the reduced stiffness in bending systems can be identified.

## 2 Theoretical background

For the determination of the influence of a deteriorated point, the differential equation of the equilibrium of forces can be set up

$$m \cdot \frac{d^2}{dt^2} w(x,t) - EJ \cdot \frac{d^4}{dt^4} w(x,t) = 0 \qquad (2)$$

where m mass in kg/m
a acceleration
EJ bending stiffness
w deformation

In an undamaged system, this equation can be solved by assuming a sinusoidal course of the deformation w. However in the damaged case, the bending stiffness decreases in the range of the damage.
To solve this equation despite the damage, the single span system is divided into the two systems A and B.

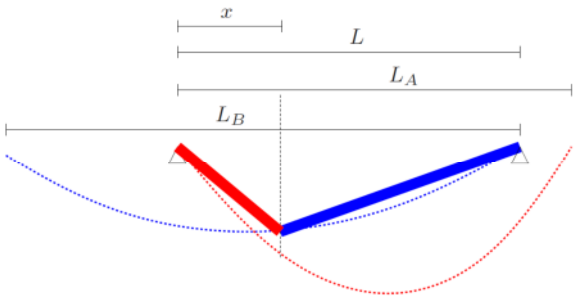



***Figure 2:*** *Subsystems*

The differential equation is solved separately for both systems. In order to connect both systems, the boundary conditions between both systems are

- equal deflection $w$ in both subsystems
- equal bending moments in both systems at the damage
- the difference of the bending angle between both systems is

$$\Delta\alpha = \frac{M}{K} \qquad (3)$$

where α bending angle between both systems
M bending moment at the damage
K bending stiffness

The bending stiffness can be determined by

$$K = \frac{EJ_{damage} \cdot EJ}{EJ - EJ_{damage}} \cdot \frac{1}{L_{damage}} \qquad (4)$$

where $K$ spring, representing the damage
$EJ_{damage}$ bending stiffness at the damage
$EJ$ undamaged bending stiffness
$L_{damage}$ length of the damage
normally assumed to be equal to the depth of the damage

The resulting equation of this procedure is given in Eq. 5 (see Fig. 3).

$$\Bigg(-\cos\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x\right)\cdot \sin\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x-\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot L\right) + \sin\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x\right)\cdot \cos\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x-\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot L\right)\Bigg) + \frac{EJ}{K}\cdot\sin\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x\right)\cdot\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega} \cdot\sin\left(\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot x-\sqrt[4]{\frac{m}{EJ}}\cdot\sqrt{\omega}\cdot L\right) = 0 \qquad (5)$$

where $m$ mass
$EJ$ bending stiffness
$\omega$ Eigen-Kreisfrequenz
$L$ length of the beam
$x$ location of the damage
$K$ spring representing the stiffness at the damage

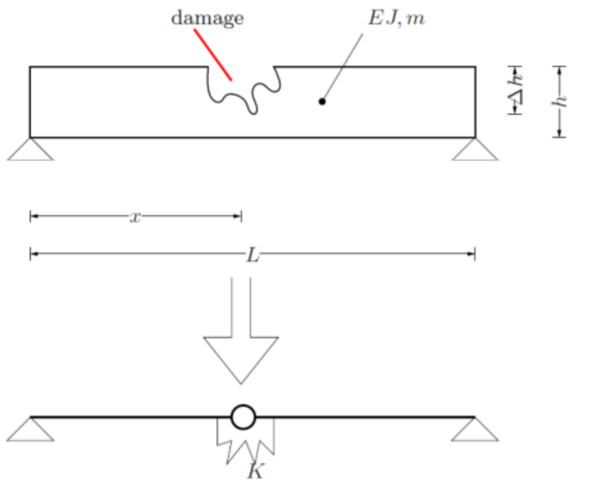
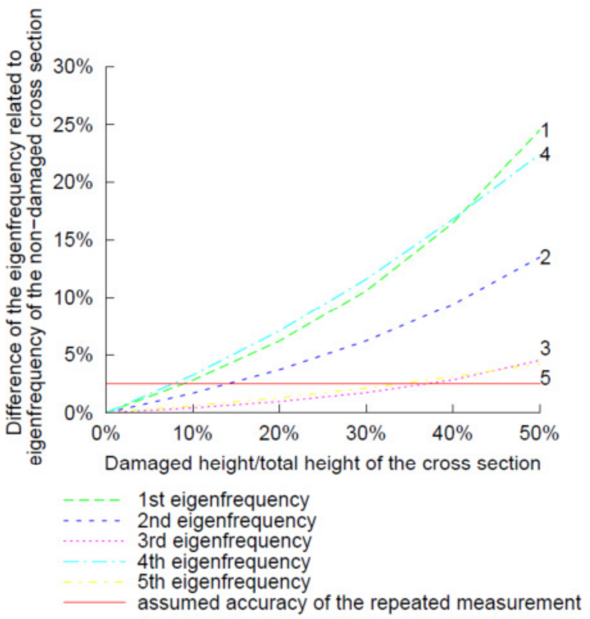



*Figure 3:* Parameters of Eq. 5

Solving this equation, the influence of a damage in the location x on the eigenfrequency can be evaluated (see Fig. 4).




Parameters

- span: 10 m
- cross section: h/b = 50/16cm
- strength class: GL24h
- mass: 200 kg/m
- damage at $x$=6.282m
- single span girder

***Figure 4:*** *Evaluated influence of a damage on the eigenfrequency*

# 3 Preliminary experimental investigations

## 3.1 General

In order to verify the derived equation, up-to-now one beam has been experimentally studied. Within this study the eigenfrequencies of a timber beam C24 were measured. The cross section is b/h = 6/10 cm, the beam length is 3,06 m (see Fig. 5) and the span is 3,00 m.

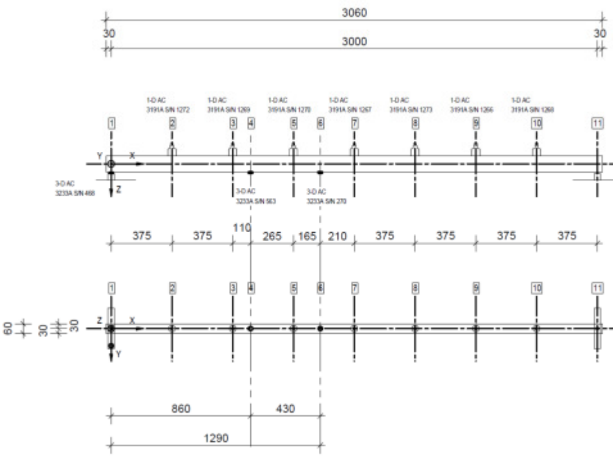



***Figure 5:*** *Drawing of the test setup*

The eigenfrequency was measured by 7 vertical accelerometers, uniformly distributed along the beam axis (see Fig. 6).

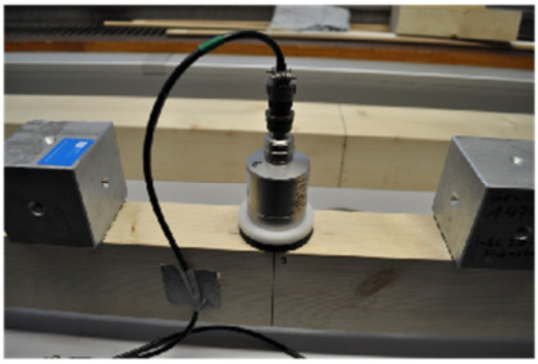


***Figure 6:*** *Accelerometer*

In order to simulate a damage of the beam, the timber cross section in midspan was cutted sequentially within 5 mm steps until a total "damage" of 55mm (see Fig. 7).

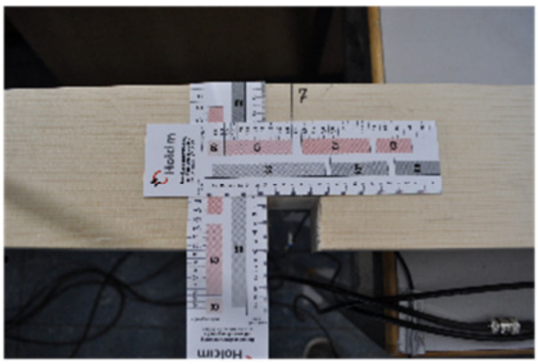


***Figure 7:*** *"Damage" of the beam in midspan (here 30% of the cross section height)*

In order to identify the non geometrical parameters as bending stiffness and mass, the eigenfrequencies were determined with and without an additional mass. This mass was applied by cubes of alloy uniformly distributed along the beam axis as can be seen in Fig. 8.

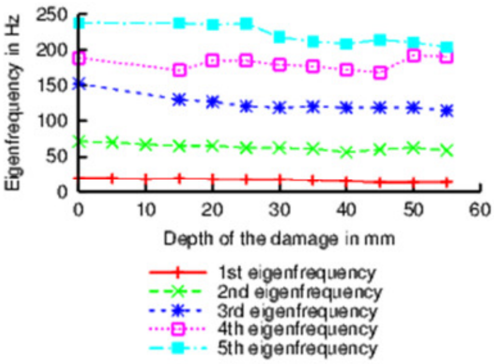
***Figure 8:*** *Beam B1 loaded with an additional mass*

The excitation of the system is ambient and the duration of one set of measurements of the vibrations lasts 20 minutes.

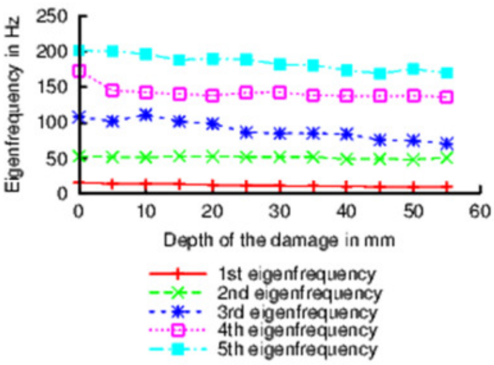
### 3.2 Influence of the "damage" on the eigenfrequency

The damage in the cross section was simulated by a cut in midspan with different depths. The eigenfrequency was measured for every depth within 5mm steps. A modal analysis was performed at every 5 mm depth of the cut.

In Fig. 9 the measured eigenfrequencies for different depths of the damage are shown.




(a) unloaded system

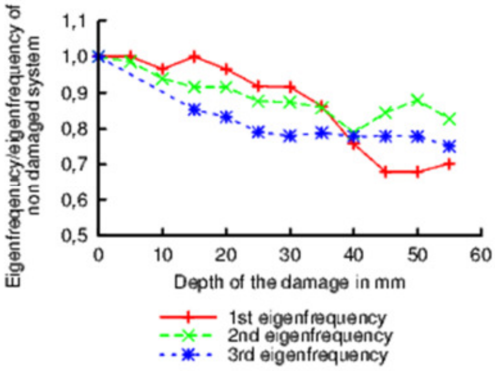



(b) loaded system

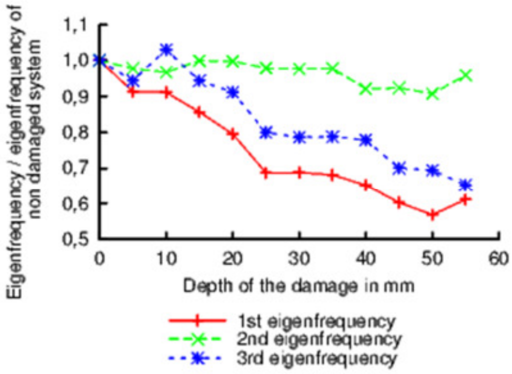
***Figure 9:*** *Measured eigenfrequencies with different depths of the "damage"*

As can be seen, the measured eigenfrequencies are in the range of 10Hz to 200Hz. Since the accuracy of the results of the measurements drops with increasing value, only the eigenfrequencies below 150Hz are considered in the following.

In Fig. 10 the eigenfrequencies are related to the eigenfrequencies of the non damaged system.




(a) unloaded system




(b) loaded system

***Figure 10:*** *Ratio of the eigenfrequencies of the beam at different stages of the damage related to the eigenfrequency of the non damaged system*

As can be seen, the eigenfrequencies drop within increasing size of the damage (see Tab. 1).

***Table 1:*** *Ratio of the eigenfrequency depending on the depth of the damage*

| | eigenfrequency | | |
|---|---|---|---|
| depth | 1st | 2nd | 3rd |
| unloaded beam | | | |
| 20mm | 0.97 | 0.92 | 0.83 |
| 40mm | 0.75 | 0.79 | 0.78 |
| 55mm | 0.70 | 0.83 | 0.75 |
| loaded beam | | | |
| 20mm | 0.80 | 0.99 | 0.91 |
| 40mm | 0.65 | 0.92 | 0.77 |
| 55mm | 0.61 | 0.95 | 0.65 |

A significant decrease of the first eigenfrequency of more than 3% can be measured for a damage of the unloaded beam, and more than 20% for the loaded beam.

However the 2nd frequency drops although the damage is in midspan and should not affect the second eigenfrequency, since the eigenmode leads to a value =0 at midspan (see Fig. 11).

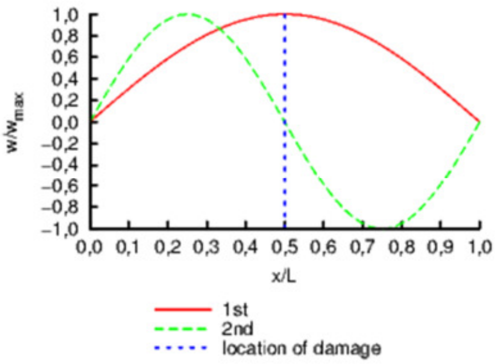



***Figure 11:*** *1st and 2nd eigenmode of the measured beam*

Therefore no bending moment should exist in midspan. Up-to-now the reason for this influence is not known, however additional tests – e.g. determination of the MoE – will be performed on this type of beams in order to identify the reason for this behaviour.

Generally spoken, the eigenfrequencies are influenced by the depth of the damage. So it should be possible to identify damages at a certain stage by comparing the eigenfrequencies between those stages. One focus of the ongoing studies will be the clarification, whether the damage can be recognized and which accuracy of the measurements is required in order to predict the damage in time.

# 4 Comparison between measurement and evaluation

## 4.1 Input values

For the determination of the eigenfrequency geometrical parameters as well as parameters related to the cross section need to be known. The geometrical values can be easily measured, whereas the bending stiffness as well as the mass can often not be directly measured. For that reason cubes made of alloy were placed on the beam and the eigenfrequency was measured for the loaded configuration. Based on

$$f = \frac{\pi}{2 \cdot L^2} \cdot \sqrt{\frac{EJ}{m}} \tag{6}$$

the influence of the additional mass Δm on the frequency can be determined by

$$m = \frac{1}{\left(\frac{f_1^2}{f_2^2} - 1\right)} \cdot \Delta m \tag{7}$$

where $m$ unknown mass of the specimen
$f_1$ frequency of beam without additional mass
$f_2$ frequency of beam with additional mass
$\Delta m$ applied additional mass

However the applied load is not a uniform distributed load as required in the equation, but consists of single loads. In order to consider these single loads, the energy of these loads in dependency on the eigenmode is determined. The value of the uniform distributed load is determined by the equality of the energy of the single eigenmode.
With the measured first eigenfrequencies of the loaded and the non loaded system, the mass of the beam can be determined to 2,86kg/m. The weighing of the beam leads to a mass of 3,08kg/m, which leads to a difference of 9%.

***Table 2:*** *Measured first eigenfrequency of the beam*

| | |
|---|---|
| without additional mass | 18.707Hz |
| with additional mass Δm | 14.437Hz |

Applying this determined weight and the frequency of the non loaded system in Eq. 6, a MoE of 12914 N/mm² can be determined by

$$EJ = \frac{4 \cdot L^4}{\pi^2} \cdot \frac{f_1^2 \cdot f_2^2}{f_1^2 - f_2^2} \cdot \Delta m \tag{8}$$

This MoE has not been verified experimentally so far, however this parameter will be determined by a four-point bending test at the end of the test series.

## 4.2 Numerical simulation of the damage

With the input values, the eigenfrequencies of the beam with different depths of damage have been determined by solving Eq. 5. As can be seen in Fig. 12 the evaluation according to Eq. 5 overestimates the results.

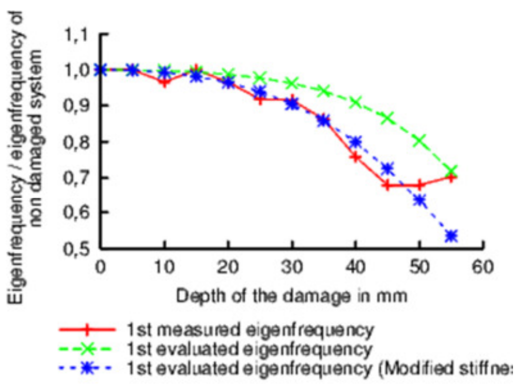



***Figure 12:*** *Comparison between test and evaluation of the 1st eigenfrequency of the unloaded system*

The modelling of the damage is one important parameter. According to Eq. 4 the stiffness of the cross section is fully active until the edge of the damage. In the range of the damage it suddenly drops to the stiffness of the reduced cross section. So no transition is considered. Therefore it is expected, that the stiffness representing the damage according to Eq. 4 is overestimated.

To consider the transition from the damage to the non damaged range, it is assumed that the stresses are spread and the damaged depth decreases in a ratio depth:length equal 1:2 parallel to the grain, so the effective cross section increases continuously (see Fig. 13).

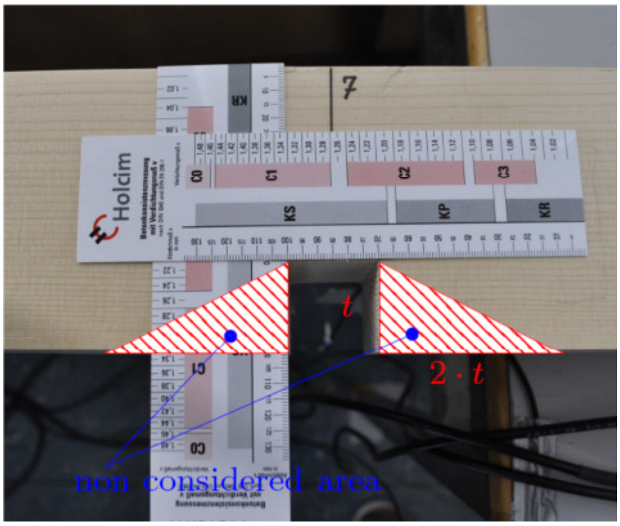



***Figure 13:*** *Reduced cross section for the determination of the effective stiffness representing a damage*

If the system is loaded, differences even in the first eigenfrequency occur (see Fig. 14).

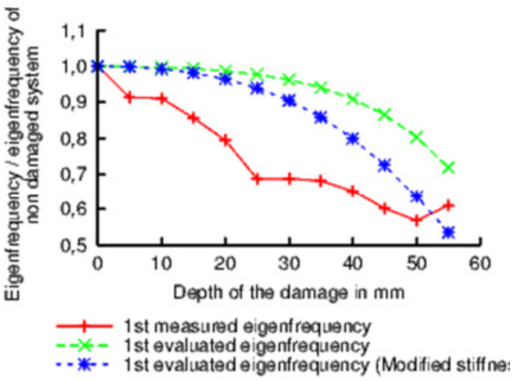



***Figure 14:*** *Comparison between test and evaluation of the 1st eigenfrequency of the loaded system*

At the moment no systematic error can be identified, since the influence of the 3rd eigenfrequency of the loaded system can be modelled with a sufficient accuracy (see Fig. 15) in difference to the 1st eigenfrequency (see Fig. 14).

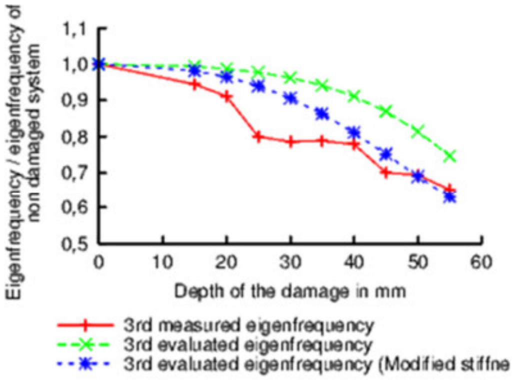



***Figure 15:*** *Comparison between test and evaluation of the 3rd eigenfrequency of the loaded system*

However the tendency of the reduction of the eigenfrequency depending on the depth of the damage can be seen. One influence on the difference between the evaluation and the measurements is, that the measured eigenfrequency drops from the non-damaged system to a damage size of 5mm. The evaluation does not show this dropping of the frequency between those two states. However the non damaged status was used in order to identify the parameters for the evaluation (see Eq. 7 and Eq. 8).

The behaviour of the eigenfrequency can be modelled even if there are larger differences. Within the shown results of one test the evaluation depends less on the damage depth than the measurements. Therefore it is expected, that the monitoring of the eigenfrequency could be one tool in order to identify damages in an early stage. However within the ongoing studies these preliminary conclusions will be validated against additional tests.

## 5 Reverse identification of the damage

The original goal of these studies was the identification of existing damages without the knowledge of the bending stiffness as well as the mass.

Since there are 4 unknown parameters in Eq. 5 as

- mass m
- bending stiffness EJ and especially the MoE
- location of the damage x
- depth of the damage

four solutions of Eq. 5 and four eigenfrequencies, respectively, need to be known. However in Eq. 5 the mass as well as the bending stiffness are part of a fraction, so the values themselves cannot be determined directly by a numerical solution process. However the bending stiffness needs to be known separately from the mass. For that reason, one eigenfrequency has to be considered with an additional mass.

Within the solution process the parameters are adjusted until the measured eigenfrequencies fit to the evaluated eigenfrequencies. If the parameters of Eq. 5 are determined by means of a "normal" Newton-iteration, the parameters are adjusted until the error is negligible.

However Eq. 5 has several zero-crossings. These zero-crossings represent the eigenfrequencies (see vertical lines in Fig. 16).
A "normal" Newton-iteration could solve the equation at the wrong eigenfrequency. For that reason, the numerical solution by means of the Newton-iteration was modified, that it first checks the number of the eigenfrequency, determines the values and the numerical derivations at the regarded point.

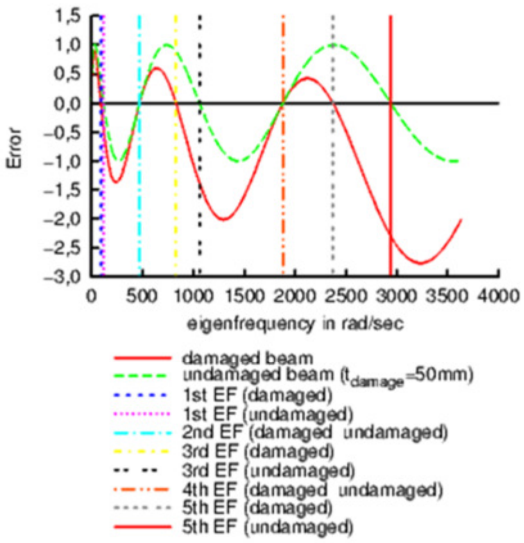



***Figure 16:*** *Error of Eq. 5*

Nevertheless the solution process has some limitations:

- The crossing of the value x over the peaks of the eigenmode, related to the used eigenfrequencies, is not possible in the current version of the iteration process. Therefore the starting point of the value x should be chosen in between the peaks and several iterations are necessary.
- Due to the symmetry of the eigenmodes related to midspan, only the distance from midspan to the position of the damage x can be determined. It is not possible, to determine, whether the damage is on the left hand side or on the right hand side of the midspan. Nevertheless it should be possible to identify, whether a damage exists or not.

Beside these limitations the exactness of the measured values can influence the reverse identification. As shown in Fig. 12 to Fig. 15 there are differences between the evaluation and the measured values (see Tab. 3).

***Table 3:*** *Ratio between the measured and the evaluated eigenfrequencies*

| | eigenfrequency 1st | eigenfrequency 2nd |
|---|---|---|
| unloaded | 0.94 | 0.75 |
| loaded | 0.81 | 0.84 |

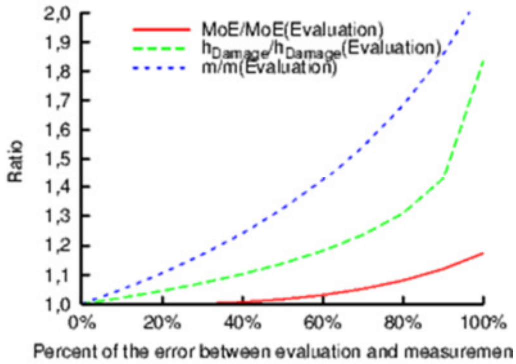



***Figure 17:*** *Influence of the differences between the evaluation and the measurements on the determined parameters*

As can be seen the MoE is hardly influenced by the derivation of the input values, whereas the mass is influenced the most. Concerning the thickness in the range of the damage, the evaluated thickness is influenced the most between 90% and 100% of the existing differences. So even the thickness of the cross section in the range of the damage can be determined in a sufficient way except for range between 90% and 100% of the differences between the measurements and the evaluation. This comparison is not a general conclusion, but it shows, that the differences may affect the results.

## 6 Conclusion and outlook

Within the study it can be shown, that in principle a damage can be determined by the monitoring of the eigenfrequencies as long as the damage is within a certain range. So this method could have the capability for the monitoring of bridges in order to detect damages in an early stage.

However several parameters could influence the results of the measurements and therefore the drawn conclusions. So within the scope of the ongoing studies following points will be studied in detail among others:

- How large is the influence of the accuracy of the measurements, if the equipment is assembled and after the measurements demounted?
- What is the impact of the changing surrounding conditions mainly the moisture content of the timber at different points in time within a year?
- Which method for the excitation of the system is the best and most suitable for the practical application?
- Are the studies transferable to other structural systems than a single span girder?
- What is the effect of the equipment of the bridge as guardrail on the eigenfrequencies and the drawn conclusions respectively?
- Can the damage be identified early enough, or is the influence of the damage on the eigenfrequency too low to be detected in time?

If these questions are solved within the ongoing studies, the measurements of the eigenfrequencies might be one possibility for the examination of elements subjected to bending as e.g. bridges. One advantage of this method would be, that the results of the assessment of structures will depend less on the experience of the examiner as it is today.

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